

EXTRA MATHEMATICS ADMISSIONS TEST

December 2021

Time allowed: 1 hour

Surname	
Other names	

This paper contains 10 multiple choice questions.

Calculators are not permitted.

For each question on pages 2–11 you will be given **five** possible answers, just one of which is correct. Indicate for each question **A–J** which answer (a), (b), (c), (d), or (e) you think is correct with a tick (✓) in the corresponding column in the table below.

	(a)	(b)	(c)	(d)	(e)
A					
B					
C					
D					
E					
F					
G					
H					
I					
J					

A. Which of the following expressions has the largest value? Note that all angles are given in degrees.

(a) $\cos(10^\circ)$,

(b) $\sin(115^\circ)$,

(c) $\cos(375^\circ)$,

(d) $\sin(85^\circ)$,

(e) $\cos(-20^\circ)$.

Please turn over

B. In the expansion of $(x^2 + xy + y^2)^n$, where n is a positive whole number, the coefficient of x^3y^{2n-3} is

(a) $\binom{n}{3}$

(b) $\binom{n}{3} \times \binom{n}{2}$

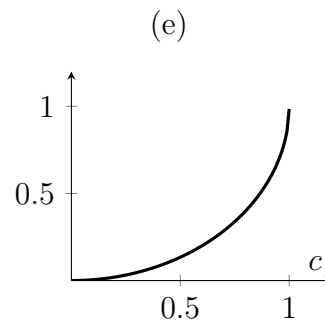
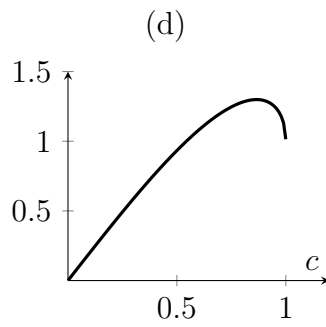
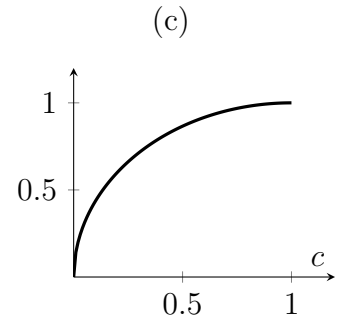
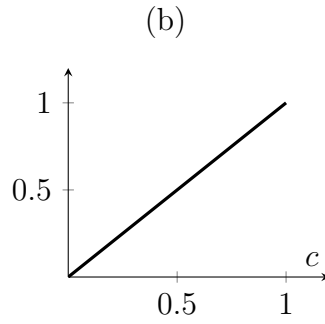
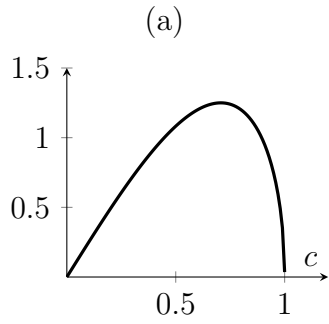
(c) $\binom{n}{3} + 2 \times \binom{n}{2}$

(d) $2 \times \binom{n}{2}$

(e) $\binom{n}{3} + \binom{n}{2}$

Please turn over

C. Given a real number c with $0 < c < 1$, the line $y = c$ intersects the circle $x^2 + y^2 = 1$ at two points. These two points, together with $(1, 0)$ and $(-1, 0)$, form a quadrilateral. Which of the following graphs is a plot of the area of that quadrilateral against c ?



Please turn over

D. A particle moves along the x -axis. At time $t = 0$ the particle starts at $(0, 0)$ with initial speed 1, moving towards $x = 1$. When the particle reaches $x = n$ for any positive integer n , its speed immediately changes to 2^{-n} but its direction is unchanged. What is the particle's position at time $t = 100$?

(a) $x = \frac{89}{16}$,

(b) $x = \frac{105}{16}$,

(c) $x = \frac{3200}{32}$,

(d) $x = \frac{421}{64}$,

(e) The particle has escaped to infinity.

Please turn over

E. The polynomial equation $x^4 - (2k + 1)x^2 + 2x + k^2 - 1 = 0$ has exactly four real solutions x if and only if

(a) $k > 1$,

(b) $k > -\frac{5}{4}$,

(c) $k > \frac{3}{4}$,

(d) $k < -\frac{5}{4}$ or $k > \frac{3}{4}$,

(e) $\frac{3}{4} < k < 1$ or $k > 1$.

Please turn over

F. The point A has coordinates $(3, 4)$. The origin $(0, 0)$ and the point A both lie on the circumference of a circle \mathcal{C} . The diameter of \mathcal{C} through A also meets \mathcal{C} at another point B . The distance between B and the origin is 10. It follows that the coordinates of B could be either

(a) $(-5\sqrt{2}, 5\sqrt{2})$ or $(5\sqrt{2}, -5\sqrt{2})$,

(b) $(-4, 3)$ or $(4, -3)$,

(c) $(-5, 5\sqrt{3})$ or $(5, -5\sqrt{3})$,

(d) $(-8, 6)$ or $(8, -6)$,

(e) $(-5\sqrt{3}, 5)$ or $(5\sqrt{3}, -5)$.

Please turn over

G. Without calculating it directly, which of the following numbers is the square of 123,456,789?

(a) 15,241,578,710,190,521,

(b) 15,241,578,730,190,521,

(c) 15,241,578,750,190,521,

(d) 15,241,578,770,190,521,

(e) 15,241,578,790,190,521.

Please turn over

H. A function $f(x)$ satisfies the following equation

$$f(x) + f(y) = \frac{1}{f(xy)}$$

for any real positive numbers x and y , and also satisfies $f(x) > 0$ for all real positive numbers x . It follows that $f(2021)$ is

- (a) 1,
- (b) 2021,
- (c) $\log_e 2021$,
- (d) $\frac{1}{\sqrt{2}}$,
- (e) $\frac{1}{\log_e 2021}$.

[Hint: try substituting $x = 1$ and $y = 1$ into the given expression.]

Please turn over

I. Given that there are positive real numbers a, b, c that satisfy

$$\int_a^b \log_c (\sin^4 x \tan^2 x) \, dx = 1 \quad \text{and} \quad \int_a^b \log_c (\sin^2 x \cos^2 x) \, dx = 3,$$

it follows that the value of

$$\int_a^b \log_c (\sin^4 x \cos^2 x) \, dx$$

must be equal to

- (a) 4,
- (b) 5,
- (c) 6,
- (d) 7,
- (e) 8.

[Note that $\sin^4 x$ means $(\sin x)^4$.]

Please turn over

J. There is a straight line that is normal to the curve $y = x^3 - kx$ at two different points if and only if

(a) $k \geq \sqrt{3}$,

(b) $k^2 \geq 3$,

(c) $k^2 \geq 1$,

(d) $k \geq 1$,

(e) $k \geq \sqrt{3}$ or $k \leq -1$.

End of last question